Technical Notes

Boubaker Polynomials Expansion Scheme-Related Heat Transfer Investigation Inside Keyhole Model

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Nomenclature

b = Gaussian beam radius, m
h = keyhole height, m
N_0 = prefixed integer
P_{ak} = total power absorbed, J
Q_v = power per unit volume, J m^{-3}
T = absolute temperature, K
T_0 = maximum absolute temperature, K
T_∞ = room absolute temperature, K
V = keyhole volume, m^3
α_n = Boubaker polynomials minimal positive roots, dimensionless
ξ_q = real coefficients, dimensionless

I. Introduction

IN THE last decades, various numerical techniques have been used to model the laser welding to solve the heat transfer equations [1–3]. Among many models, the laser welding keyhole (Fig. 1) model was proposed and discussed in depth [4–6].

In this note, we tried to set a cylindrical model as a guide to solve the heat equation inside a laser welding keyhole model and derive the cooling velocity.

II. Theory

The main heat equation inside the keyhole (Fig. 1) is

$$\begin{cases} \frac{\partial T(x,t)}{\partial t} = \frac{1}{D}\frac{\partial^2 T(x,t)}{\partial x^2}, \quad t > 0, \quad |x| < b \\ T(x,t)|_{t=0} = T_0 \times e^{-\frac{x^2}{2b^2}}, \quad T(x,t)|_{t\to\infty} = T_\infty \end{cases} \tag{1}$$

where T_∞ is the room temperature.

The power Q_v per unit volume absorbed by the keyhole is calculated as a guide to determine the maximal central temperature T_0:

$$Q_v = \frac{P_{\text{ak}}}{V_{\text{keyhole}}} = \frac{4P_{\text{ak}}}{h\pi b^2} \tag{2}$$

where P_{ak} is the total power absorbed by the keyhole volume.

The expression of $T(x,t)$ is expressed as an infinite sum of the Boubaker polynomials [7–11], for which the expression fits the boundary condition:

$$T(x,t) = T_0 \times e^{-\frac{x^2}{2b^2}} \times \frac{1}{2N_0}\sum_{n=1}^{N_0} \xi_n \cdot B_{4n}\left(t\frac{\alpha_n}{t_m}\right) \tag{3}$$

Where α_n are the minimal positive roots of the Boubaker $4n$-order polynomials B_{4n} [9–11], r_m is the maximum sheet radial range (where the temperature is supposed to be room temperature), N_0 is an even given integer, T_0 is the maximal central temperature, and ξ_n are coefficients to be found.

III. Solution and Discussion

Using the Boubaker polynomials expansion scheme (BPES) [9–11], and thanks to Boubaker polynomials' properties, a solution to Eq. (1) is proposed (Fig. 2). In several cases, the cooling velocity profile is an efficient guide to determine interesting characteristics. DERIVE_6 software‡ has been used to evaluate the cooling velocity profile (Fig. 3).

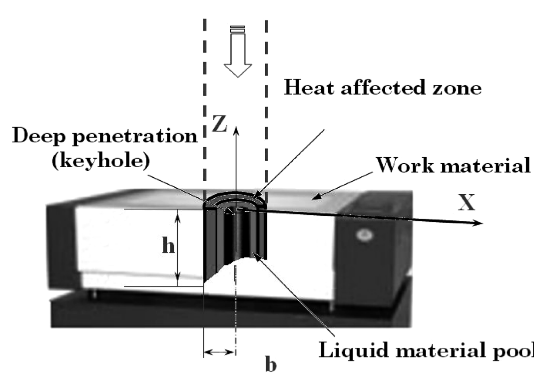

Fig. 1 Studied model.

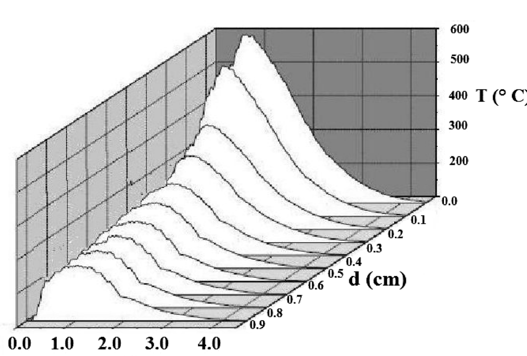

Fig. 2 Temperature 3-D profile.

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‡Data available at http://derive-europe.com.

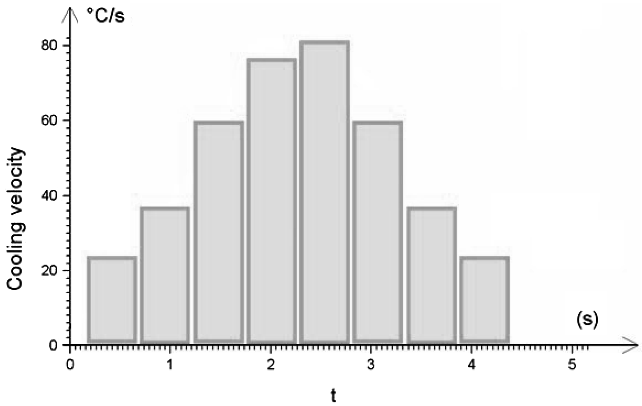

Fig. 3 Cooling velocity profile.

IV. Conclusions

In this Note, a model of heat transfer inside a cylindrical keyhole laser welding is presented. The BPES protocol led to a temperature evolution which allowed evaluation of the cooling velocity.

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